LINEAR COMBINATIONS OF MORPHOLOGICAL OPERATORS: THE MIDRANGE, PSEUDOMEDIAN, AND LOCO FILTERS

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ABSTRACT

Morphological image processing filters preserve shapes related to the structuring element shape of the operator. The basic morphological operators are minimum (erosion) and maximum (dilation) operations performed on the pixels within a structuring element. Although these operators (and the compound operators formed from them) are able to smooth noise, they also introduce a statistical and deterministic bias, which is unacceptable in some applications. However, since every morphological operator has a complementary operator that is equally and oppositely biased, we propose averaging the complementary operators to alleviate the bias. Of the three filters formed by averaging the standard morphological operators, two are the previously-defined midrange filter and pseudomedian filter, while one is a new filter, which we call the LOCO filter. Under most conditions, the LOCO filter is the best of these at reducing impulses and noise.

1. INTRODUCTION

The techniques of mathematical morphology provide shape-based methods for image processing. The basic morphological operators have been shown to be effective at reducing various types of noise while preserving shapes compatible with the structuring element of the operator. However, the basic morphological operators introduce a statistical and deterministic bias to signals that they process [1-3]. For many applications, such as segmentation, this bias is not a problem. However, in applications where preservation of intensity levels is important, such as quantitative infrared thermography, biased morphological operators may not be used to process images.

Since morphological operators are defined in complementary pairs that are equally and oppositely biased, one potential solution to the biasing problem is to average the complementary operators. We show in this paper that of the three filters formed by taking the averages of complementary morphological operators, two of them are equivalent to the previously-defined midrange and pseudomedian filters [4-6], while one is an entirely new filter. This new filter, which we call the LOCO filter, is the best of the three at reducing most types of noise, especially impulse noise.

2. MORPHOLOGICAL FILTERS

The two basic morphological operators are dilation and erosion. Let f($\mathbf{x}$) denote an n-dimensional function (the image) and N denote a compact k-dimensional set (the structuring element) with k = n. Also, let $\tilde{N}$ denote the 180° rotation of N, and let the translation of the set N by a point $\mathbf{z}$ be denoted by a subscript: $N_{\mathbf{z}}$. The morphological erosion of f by N is defined by the following equation:

$$(f \ominus \tilde{N}) (\mathbf{x}) = \inf_{\mathbf{y} \in N_{\mathbf{x}}} \{ f(\mathbf{y}) \} \qquad (1)$$

Morphological dilation is defined by:

$$(f \oplus \tilde{N}) (\mathbf{x}) = \sup_{\mathbf{y} \in N_{\mathbf{x}}} \{ f(\mathbf{y}) \} \qquad (2)$$

For discrete images and structuring elements, the infimum (inf) and supremum (sup) are equivalent to the minimum and maximum, respectively. Because these operators output extreme order statistics, it is obvious that they introduce statistical and deterministic bias to the functions.

Morphological opening is a compound operator consisting of erosion followed by dilation; similarly, morphological closing is dilation followed by erosion. Opening is defined by:

$$\text{Open}\{f(\mathbf{x}); N\} = [(f \ominus \tilde{N}) \oplus N] (\mathbf{x}). \qquad (3)$$

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Closing is defined by:

$$\text{Close}\{f(\mathbf{x}); N\} = [\,(\,f \oplus \tilde{N}\,)\,\Theta\, N\,]\,(\mathbf{x}). \qquad (4)$$

The doubly compound morphological operators open-close (OC) and close-open (CO) are defined as opening followed by closing and as closing followed by opening, respectively:

$$\text{OC}\,\{f(\mathbf{x}); N\} = \text{Close}\{\text{Open}\{f(\mathbf{x}); N\}; N\} \qquad (5)$$

$$\text{CO}\,\{f(\mathbf{x}); N\} = \text{Open}\{\text{Close}\{f(\mathbf{x}); N\}; N\} \qquad (6)$$

Maragos and Schafer [2, 3] have demonstrated the deterministic bias introduced by the morphological operators by proving the following inequalities:

$$(f\,\Theta\,\tilde{N}) = \text{Open}\{f; N\} = f = \text{Close}\{f; N\} = (f\,H\,\tilde{N}) \qquad (7)$$

$$\text{OC}\,(f; N) = \text{med}^{\infty}\,(f; W) = \text{CO}\,(f; N), \qquad (8)$$

where $\text{med}^{\infty}\,(f; W)$ denotes the median root signal (that is, a signal invariant to further median filtering) achieved by repeatedly median filtering $f(\mathbf{x})$ with window $W = N\ H\ \tilde{N}$. Stevenson and Arce [1] illustrate these properties statistically by deriving the distribution function of the output of the CO operator, which is biased toward higher values than the input distribution. The output distribution of OC is analogous to CO, but biased toward smaller values. The other morphological operators have more severely biased distributions, as would be expected from (7) and (8).

3. LINEAR COMBINATIONS OF MORPHOLOGICAL FILTERS

One potential solution to the bias problems of the basic morphological operators is to form new filters that take the average the complementary operators. The three filters formed by this method are the midrange, pseudomedian, and LOCO filters. They are described in more detail below.

3.1. Midrange Filter

Morphological erosion simply returns the minimum value within its structuring element, while morphological dilation returns the maximum value. The average of the erosion and dilation is therefore the midpoint of the range of values in the structuring element. This is the midrange filter [6, 7], a well-known estimator in the theory of order statistics. It is the minimum variance unbiased estimator of the median of a uniform noise distribution [7].

The response of the midrange filter is typically not desirable for image processing, because it destroys edges. Its performance is good for constant signals in the presence of uniformly distributed noise; however, for other noise distributions and for images with edges, other filters perform better.

3.2. Pseudomedian Filter

The pseudomedian filter was first developed by Pratt, Cooper, and Kabir [4] in 1985 to mimic the response of the median filter. In one dimension, their definition for the pseudomedian filter of window width 5 (PMED_5) was

$$\text{PMED}_5\left\{a,\ b,\ c,\ d,\ e\right\} =$$

$$\frac{1}{2}\max\left\{\min\{a,b,c\},\ \min\{b,c,d\},\ \min\{c,d,e\}\right\}$$

$$+\,\frac{1}{2}\min\left\{\max\{a,b,c\},\ \max\{b,c,d\},\ \max\{c,d,e\}\right\}. \qquad (9)$$

Pratt [5] later called the two halves of this definition the "maximin" and "minimax" functions. Schulze and Pearce [8] defined a two-dimensional pseudomedian filter analogous to the 1-D definition (9).

Morphological opening consists of erosion followed by dilation. Erosion finds the minimum over a particular structuring element, and the dilation that follows this erosion finds the maximum of the previously-computed minima in the structuring element. Opening is thus exactly the same as the "maximin" portion of the pseudomedian definition (9). Similarly, morphological closing is dilation followed by erosion, so it finds the minimum of the maxima and is the "minimax" portion of (9). Therefore, a new definition of the pseudomedian filter (PMED) that is completely equivalent to the previous definitions is

$$\text{PMED}\left\{f(\mathbf{x}); N\right\} =$$

$$\frac{1}{2}\,\text{Open}\left\{f(\mathbf{x}); N\right\} + \frac{1}{2}\,\text{Close}\left\{f(\mathbf{x}); N\right\}, \qquad (10)$$

where $f(\mathbf{x})$ is the (n-dimensional) signal to be filtered and N is the structuring element of the morphological operators.

The response of the pseudomedian filter is indeed similar to that of the median filter, with two very important exceptions. First, the pseudomedian filter does not completely remove isolated impulses, either high-valued or low-valued, but reduces their amplitude to one-half the original values. This can be verified by noting that opening preserves negative impulses but removes positive impulses, whereas closing preserves positive impulses and removes negative impulses. Second, while the output of the median filter is restricted to values that appear in the original signal, the pseudomedian filter output may take on values that do not appear in the original signal because it takes the average of two values. This is important when working with signals that are quantized. The output of the pseudomedian filter may have to be rounded or truncated to restrict it to the same levels. Other differences between the pseudomedian and median filters, notably the effect of the structuring element shape, are described in [8]. The 1-D finite-length root signal set of the pseudomedian filter is shown to be identical to that of the median filter in [9].

3.3. LOCO Filter

Open-closing and close-opening have been shown to have many desirable properties [1-3]; for example, either operation reduces a 1-D signal to a median filter root signal in one pass. Open-closing (OC) is simply opening followed by closing, while close-opening (CO) is closing followed by opening. Unlike opening and closing, OC and CO are able to remove both positive and negative impulses. After considering the midrange filter as the

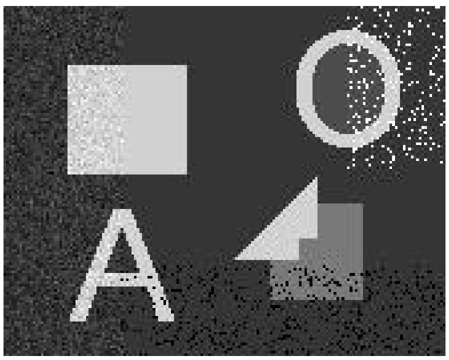

Fig. 1. Noisy original image.

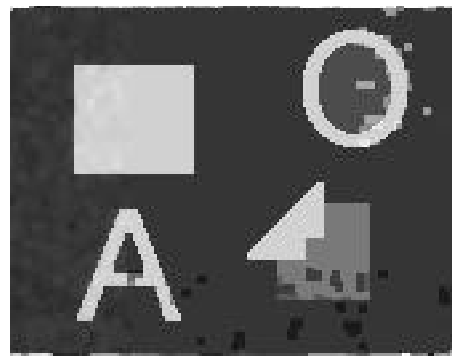

Fig. 4. LOCO-filtered image (N = 3x3 square).

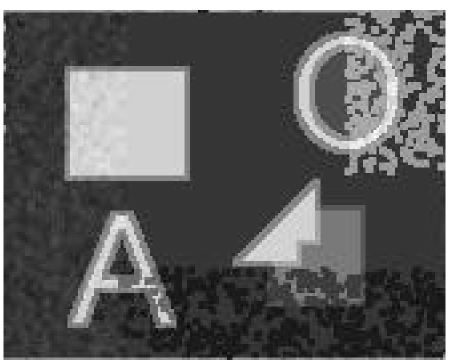

Fig. 2. Midrange-filtered image (N = 3x3 square).

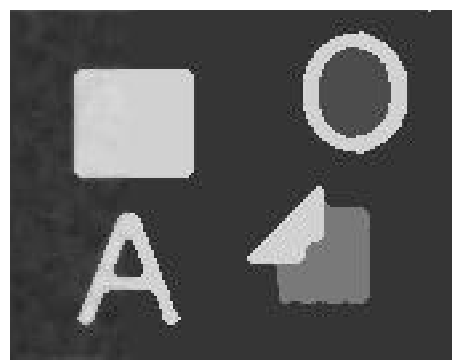

Fig. 5. Median-filtered image (5x5 square window)

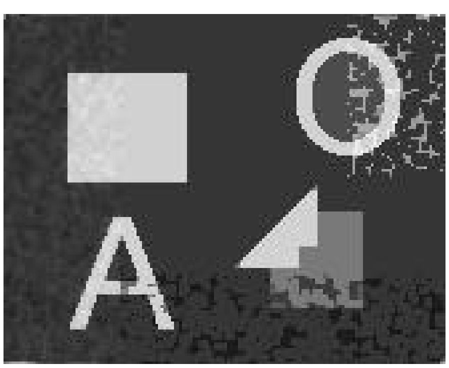

Fig. 3. Pseudomedian-filtered image (N = 3x3 square).

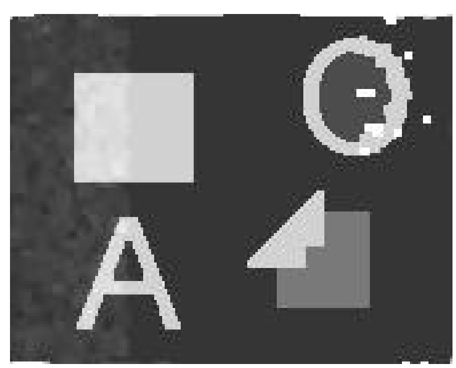

Fig. 6. Image after Close-Opening (N = 3x3 square).

average of erosion and dilation and the pseudomedian filter as the average of opening and closing, it is logical to form a filter from the average of OC and CO. We call this filter the LOCO filter, for *L*inear combination of *O*C and *CO*.

$$\mathrm{LOCO}\left\{f(\mathbf{x});\,N\right\} =$$

$$\frac{1}{2}\,\mathrm{OC}\left\{f(\mathbf{x});\,N\right\} + \frac{1}{2}\,\mathrm{CO}\left\{f(\mathbf{x});\,N\right\} \qquad (11)$$

Since both OC and CO remove positive and negative impulses, the LOCO filter is much less susceptible to impulse noise than the pseudomedian filter. In this characteristic, the LOCO filter resembles the median filter even more than the pseudomedian filter does. The LOCO filter is also not restricted to values in the original signal. Although the OC and CO individually yield a root signal in one pass, since these roots need not be identical (and usually are not), the LOCO filter does not always yield a root signal in one pass. The LOCO filter, like the pseudomedian filter, preserves edges and is not susceptible to fast-fluctuating periodic signals to which the median filter is susceptible.

4. EXAMPLES

Examples of an image filtered by linear combinations of morphological operators are given in the figures. Fig. 1 is the noisy original image, with white Gaussian noise (SNR ˜ 19 dB) on the left side, negative impulse noise (10% of pixels) at the lower central and right parts, and positive impulse noise (10%) in the upper right portion. Fig. 2 shows the results of 3x3 midrange filtering (*N* = 3x3 square). Fig. 3 is the result of 5x5 pseudomedian filtering (*N* = 3x3 square), and Fig. 4 is the result of LOCO filtering with *N* = 3x3 square. For comparison, the result of 5x5 square median filtering is shown in Fig. 5 and the result of the CO operator (*N* = 3x3 square) is shown in Fig. 6.

These figures illustrate the differences among the various linear combinations of morphological operators. The midrange filter is neither good at removing impulse noise nor at preserving edges. The pseudomedian filter preserves edges but does not reduce impulse noise or Gaussian noise very well. The LOCO filter is the best of the three at reducing impulse and Gaussian noise, although it is still somewhat susceptible to nearby impulses. The median filter (Fig. 5), in contrast, does an excellent job of removing impulse noise and has reduced the Gaussian noise quite well. However, it also distorts the object shapes in the image somewhat, particularly at sharp corners. This effect is not observed in Figs. 3 and 4 because the morphological structuring element is square. Finally, the result of CO in Fig. 6 shows the bias toward high (bright) values of this operator for both the positive impulse noise and Gaussian noise.

5. CONCLUSIONS

In this paper, we have shown how linear combinations of morphological operators may be formed to alleviate the bias introduced by the individual morphological operators. Two of the filters formed by averaging complementary operators, the midrange and pseudomedian filters, were previously defined in non-morphological terms, but the LOCO filter is a new filter. The examples given in the paper along with the known properties of the constituent operators (OC and CO) illustrate the potential superiority of the LOCO filter over the midrange and pseudomedian filters for many image processing applications.

Linear combinations of morphological operators allow the shape control of morphological filters (exerted by the selection of a structuring element) without introducing bias. For example, the LOCO filter with a square structuring element preserves 90° corners in an image while reducing noise almost as well as the square-shaped median filter, which rounds off such corners. These new filter definitions provide a way to perform transformations similar to those of mathematical morphology on images in which preservation of intensity levels is important.

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